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Dr. Jörn Epping, QUIX

Deliverable table

Deliverable no.	D6.2
Deliverable name	Set of stringent validation tests for genuine multiphoton interference
WP no.	6
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What was planned *(from Annex I:)*
D6.2: Set of stringent validation tests for genuine multiphoton interference
Set of stringent validation tests for genuine multiphoton interference. Provide algorithms which determine whether multiphoton interference takes place in the interferometer based on the measured output statistics. Losses and imperfection due to fabrication errors will be included in the analysis.

What was done.

1 Introduction

The bosonic nature of photons results in characteristic interference phenomena. The simplest such example is the Hong-Ou-Mandel test, where two perfectly indistinguishable photons entering the two input ports of a 50/50 beam splitter will always come out of it bunched in a single mode. For photonic quantum computation and other applications, it is necessary to have high indistinguishability between all photons, as distinguishable photons have a behaviour that can be simulated efficiently on a classical computer. It is thus necessary to better understand how to quantify genuine multiphoton indistinguishability, both on a theoretical footing, but also with a view to experimental certification of photon sources, and of correct bosonic behaviour in different interferometric devices.

In this report we review different approaches for the validation of multiphoton indistinguishability. As we will see, there are genuine multiphoton effects that can only be observed if all photons have a certain non-zero pairwise overlap, hence a non-zero degree of indistinguishability. These multiphoton effects are related to geometric (or "collective") photonic phases, and we will review the formalism of Bargmann invariants, which provides a unifying framework to describe multiphoton interference. We will review different approaches to the quantification of multiphoton indistinguishability, which can be used to certify the functioning of sophisticated multiphoton interferometers of the kind developed in the PHOQUSING project. We propose to review the following approaches:

- Using suppression laws for certain events in specific interferometer designs;
- Reliance on the notion of genuine multiphoton indistinguishability;
- Measurement of Bargmann invariants;
- Bayesian methods for hypothesis testing and indistinguishability estimation;
- Methods based on machine learning for pattern recognition;
- Semi-device independent methods based on weaker assumptions on the functioning of the devices.

These approaches have various points of contact and similarities, which we will review. Some of these methods have had proof-of-principle experimental demonstrations that addressed imperfections such as photon loss and imperfect interferometer setups, which we will also briefly review. Together, the reviewed complementary methods offer ways to characterize multiphoton indistinguishability, understanding it both form a theoretical point of view, but also with respect to their relevance to the applications being pursued by the PHOQUSING project.

2.1 Suppression laws: from two-photon to multiphoton indistinguishability

The seminal Hong-Ou-Mandel (HOM) effect [Hong87] describes the behaviour of two perfectly indistinguishable photons in the simplest interferometric device: a 50/50 beam splitter (BS). Each photon is described by a spectral function, or more simply wavefunction, describing all internal degrees of freedom (e.g. time of arrival, polarization, frequency spectrum). In addition, a single dichotomic degree of freedom specifies the path, i.e. in which BS input port the photon is. For two perfectly indistinguishable photons entering different BS input ports, the quantum mechanical prediction is that the two photons will always leave the BS bunched in a single output port, due to the symmetrisation requirement for bosonic statistics. For partially distinguishable photons, imperfect bunching results, and the degree of bunching can be used to directly estimate the overlap between single-photon wavefunctions.

One approach to characterize multiphoton indistinguishability is to generalize the HOM effect, looking for output events that are strictly suppressed if the photons are perfectly indistinguishable. These so-called suppression laws apply to specific multimode interferometers with particular symmetries that lead to the destructive interference that suppresses some of the possible events. Multiphoton input/output combinations that are strictly suppressed have been characterized in interferometers described by Discrete Fourier Transform (DFT) matrices [Tich10], as well as by Sylvester matrices [Cres15, Vigg18]. In the case of Sylvester interferometers, for certain small number of modes, it was shown that the fraction of suppressed events is higher than in the DFT case. For imperfect indistinguishability, just as in the HOM effect the ideally suppressed events will happen with some probability, which can be used to characterize the degree of indistinguishability (as done experimentally for two-photon experiments in [Vigg18]).

More careful modelling is required to make sense of the probability associated with suppressed events as a quantifier of multiphoton indistinguishability. A refinement of this idea was taken up by more recent work, which showed that in a well-motivated sense, Sylvester interferometers are the best interferometer design to certify indistinguishability of two photons in general multi-mode interferometers [Vigg18b]. In that work, different interferometers were considered, in a search for the one for which the outputs of distinguishable and indistinguishable photons were as different as possible, as quantified by the total variation distance of the probability distributions corresponding to these two hypotheses. This work also pointed out other interferometer designs that were later shown to be useful for certifying genuine multiphoton indistinguishability, as we review in the next section.

2.2 Tests based on the concept of genuine multiphoton indistinguishability

Intuitively, to certify multiphoton indistinguishability we need to look for effects that can only happen if all photons are indistinguishable. As a simple example: a probability of bunching > ½ in a 50/50 beam-splitter indicates some degree of two-photon indistinguishability – what we are looking for is a suitable generalization for more than two photons.

In [Brod19], such a generalization was proposed. The basic idea is as follows. Let us assume that the source prepares single-photon states such that each pair is either perfectly identical, or mutually orthogonal, and that we can have also a statistical mixture of all such possibilities. This corresponds to a density matrix of the form:

$$\rho = c_1 \rho_{AAA\ldots A} + \sum_i c_i \rho_{\vec{S_i}}$$

where $\rho_{AAA\ldots A}$ represents a state with all photons perfectly indistinguishable, and the other states correspond to preparations with at least one pair of mutually orthogonal single-photon states. In general, there are multiple decompositions of a density matrix in the form above. Genuine multiphoton indistinguishability results when coefficient c_1 in the equation above is larger than zero for all decompositions of ρ. In [Brod19] this was shown to be a reasonably good model for single-photon sources based on parametric down-conversion, and a specific family of multimode interferometers was described such that the bunching probability at the output can be used to put bounds on c_1, and hence on multiphoton indistinguishability. An experimental demonstration demonstrating genuine 3-photon indistinguishability was also reported.

In a recent experiment performed by the CNR, UNIROMA, and CNRS nodes of PHOQUSING, a new interferometer design to measure c_1 was proposed, that in principle also works for any number of photons [Pont22]. It corresponds to an interferometer with only two beam-splitter layers, and a single non-local connection between beam-splitters (assuming an otherwise local 2D grid of beam-splitters in a planar design). The experiment conclusively demonstrated genuine indistinguishability of 4 photons, taking into account the inevitable experimental noise.

2.3 Tests based on measurements of Bargmann invariants

Projective-unitary invariants are quantities pertaining to an tuple of quantum states, and which remain invariant under application of the same unitary transformation to all states in the tuple. The simplest example is the two-state overlap $\Delta_{12}= Tr(\rho_1\rho_2)$, which reduces to $|\langle\varphi_1|\varphi_2\rangle|^2$ for the case of two pure states. As it turns out, all projective-unitary invariants can be written as functions of a particular type of n-state invariant, known as Bargmann invariant [Barg64]:

$$\Delta_{12\ldots n}= Tr(\rho_1\rho_2 \ldots \rho_n),$$

Bargmann invariants have appeared in the description of geometric phases in quantum theory, in problems such as unambiguous state discrimination, and as we shall see, also in the study of photonic indistinguishability. This is not surprising, as in all these cases we are interested in physically meaningful quantities having to do with the relative standing of all states in the tuple, being invariant if all states are rotated by the same unitary.

The relational information present in a complete set of Bargmann invariants is sufficient to completely characterize the relative arrangement of a tuple of quantum states (see [Chie16, Oszm21]). This enables one to determine, for example, the dimension of the Hilbert space spanned by the tuple, or whether the states can be simultaneously diagonalized. The characterization required for these tasks is much simpler than full state tomography, as here we are only recovering the directly relevant relational information between states, rather than fully reconstructing all states in the set, and then computing the quantities of interest. In the PHOQUSING project preprint [Oszm21], it was shown how Bargmann invariants of a tuple of general quantum states can be measured with quantum circuits that are generalizations of the SWAP test. It was also briefly sketched how Bargmann invariants can be used to characterize multiphoton indistinguishability, a question we now address.

Two-state overlaps are the simplest Bargmann invariants. Suppose we have *n* single photon states, and have measured some subset of the *n(n-1)/2* possible pairwise overlaps between them. With a judicious choice of the set of measured invariants, it is possible to obtain lower bounds for all pairwise overlaps, as shown by [Galv20, Gior20]. So, for example, it is possible to measure only *(n-1)* overlaps of one state with all others, and obtain lower bounds for the remaining overlaps, something that greatly simplifies the experimental effort required to certify indistinguishability of all photons in the set. Importantly, we recall that two-photon overlaps can be directly measured via the bunching probability of a simple Hong-Ou-Mandel test.

Multiphoton indistinguishability can also be ascertained using measurement of higher-order Bargmann invariants. As a simple example, consider a *n*-th order Bargmann invariant of *n* states in a sequence. By definition, its modulus squared is the product of all *n* successive wavefunction pairs in the sequence (wrapping up in a cycle). Measurement of this single *n*-th order invariant gives a lower bound for all overlaps in the sequence. Moreover, it is possible to combine this argument with the results described above, and presented in [Galv20], to obtain overlap bounds for all pairwise overlaps, and not just those of successive states in the sequence.

Higher-order Bargmann invariants can be measured in a photonic setting in a number of ways. In [Mens17], a 3-mode QFT interferometer was used to measure the 3rd order Bargmann invariant of 3 single-photon states. It was shown that different values of this invariant can be reached, having the same, fixed value for the 3 pairwise overlaps, showing experimentally that this is indeed a collective property of the 3 photons, not accessible by two-photon experiments. In [Jone20], a similar demonstration was made with a particular balanced 4-mode interferometer, used to measure a 4^{th}-order Bargmann invariant of 4 single-photon states. In the recent PHOQUSING preprint [Pont22], an interferometer design capable of measuring the genuine indistinguishability coefficient c_1 was proposed. As it turns out, the same design is also capable of measuring Bargmann invariants of any order, something that the PHOQUSING project aims to explore for the goal of certifying multiphoton indistinguishability. PHOQUSING continues to study alternative setups for measurement of Bargmann invariants, with a view to minimizing photon loss, and the overhead due to the necessary postselection in different interferometric schemes.

2.4 Bayesian methods for hypothesis testing and indistinguishability estimation

Bayesian approaches can be employed within the validation task of multiphoton interference in linear interferometers. Such methods are based on Bayes' theorem for conditional probabilities, and on encoding the degree of belief on a given hypothesis using a probability distribution.

In the context of Boson Sampling, these methods have been introduced [Bent14] as a tool for binary hypothesis testing. In particular, let us consider the case where we aim at identifying whether H_1 (indistinguishable photons) or H_2 (distinguishable particles) provide a more accurate description of a given data sample. If no a-priori information is available, the initial belief is encoded as $p(H_1) = p(H_2) = 0.5$. These probabilities are progressively updated via the Bayes' theorem for each event $t^{(k)}$ sampled by the apparatus as

$$p^{(k)}\left(H_i\middle|\{t^{(k)}\}\right) = p^{(k-1)}\left(H_i\middle|\{t^{(k-1)}\}\right) p\left(t^{(k)}\middle|H_i\right)/N,$$

where $p^{(k)}\left(H_i\middle|\{t^{(k)}\}\right)$ is the confidence probability of hypothesis H_i after k events, $p^{(0)}(H_i) = p(H_i)$ at step 0 is the prior, N is a normalization constant, and $p\left(t^{(k)}\middle|H_i\right)$ is the probability of obtaining outcome $t^{(k)}$ for hypothesis H_i. As an example, for indistinguishable Fock state input, $p\left(t^{(k)}\middle|H_i\right)$ is provided by the permanent formula for Boson Sampling. After k events are sampled, $p^{(k)}\left(H_i\middle|\{t^{(k)}\}\right)$ thus represents the current degree of belief (between the two considered hypotheses) that the measured samples belongs to H_i.

In general, this class of binary hypothesis testing based on Bayes' theorem can be performed by considering as H_2 different classically simulable mock-up distributions, where distinguishable particles represent a notable example. A related test, derived in a similar spirit from Bayesian approaches, has been also employed in recent large-scale implementations of Gaussian Boson Sampling [Zhon20, Zhon21].

Advantages of this Bayesian hypothesis testing are the moderate number of data necessary to reach high confidence discrimination values, as shown in Refs. [Bent14, Flam20] with respect to the distinguishable particle case. Conversely, application of this test requires calculation of potentially computationally complex quantities, such as $p\left(t^{(k)}\middle|H_1\right)$ for Fock input state using the permanent formula, that does not exhibit a favourable scaling for large photon number.

Bayesian methods are also currently employed in quantum sensing for parameter estimation, both in the single-parameter and in the multi-parameter regime, since they represent a convenient choice of an unbiased estimator. In this case, the discrimination process is not limited to a set of two possible hypotheses, while the aim is to obtain estimation of one or more unknown parameters. Let us discuss the single-parameter case. Here, Bayes' theorem is applied to a continuous parameter x, an example being provided by an optical phase $\varphi \in [0,2\pi)$ between two arms of an interferometer. Starting from the prior distribution $p(x)$, i.e., the probability density function (p.d.f.) expressing the knowledge of the parameter before the experiment, Bayes' theorem updates it as:

$$p^{(k)}\left(x\middle|\left\{t^{(k)}\right\}\right) = p^{(k-1)}\left(x\middle|\left\{t^{(k-1)}\right\}\right) p\left(t^{(k)}\middle|x\right)/N,$$

where $p^{(k)}\left(x\middle|\left\{t^{(k)}\right\}\right)$ is the conditional p.d.f. after k events, $p^{(0)}(x) = p(x)$ at step 0 is the prior, N is a normalization constant, and $p\left(t^{(k)}\middle|x\right)$ is the probability of obtaining outcome $t^{(k)}$ for a given value of x (also called likelihood). After k measurements, the conditional p.d.f. expresses the update knowledge on the parameter. An estimate x_{est} and its related confidence interval $\sigma(x)$ for the parameter can be provided by the mean and the variance of $p^{(k)}\left(x\middle|\left\{t^{(k)}\right\}\right)$:

$$x_{est} = \int x\, p^{(k)}\left(x\middle|\left\{t^{(k)}\right\}\right) dx\,;$$

$$\sigma^2(x) = \int (x - x_{est})^2\, p^{(k)}\left(x\middle|\left\{t^{(k)}\right\}\right) dx.$$

Such approach has been introduced in [Vigg18b] for Bayesian validation of multiphoton interference. In general, its adoption can be thus employed by considering as parameter the pairwise indistinguishability x between multiphoton input states [Rene18]. Furthermore, we observe that Bayesian estimation is related to the maximum likelihood approach for validation discussed in [Rene21].

The Bayesian approach provides an unbiased estimation. Furthermore, such estimator is asymptotically optimal and it permits to estimate both the parameter x_{est} and a confidence interval $\sigma(x)$ without requiring multiple repetitions of the experiment. However, as for the hypothesis testing scenario, its application requires the evaluation of the likelihood $p\left(t^{(k)}\middle|x\right)$, which in the Boson Sampling case can be related to the calculation of matrix permanents. Hence, it is associated to an exponentially increasing computational effort as a function of the number of particles n.

A relevant aspect is that, if one is interested in estimating x in a decoupled fashion with respect to other noise sources (unitary matrix errors, losses), it is necessary to obtain an a-priori characterization of these parameters. Indeed, such effects need to be included in the likelihood. If these sources of noise are not considered in the likelihood estimation, this will result in an estimation of the parameter x that is typically biased towards smaller values.

2.5 Methods based on machine learning for pattern recognition

Recently, machine learning approaches have been suggested as a tool for validation of multiphoton interference in the Boson Sampling context. In particular, in Ref. [Agre19] it has been proposed and verified experimentally that pattern recognition techniques can be employed as a tool to discriminate between Boson Samplers using indistinguishable bosons or distinguishable particles. This is done by considering the scenario where one has two different data samples originated from the same unitary matrix U, and aims at determining whether these two samples are compatible (namely, it is likely that they have been generated by the same class of input states). One of the two samples is trusted, for instance obtained from a known previously certified device, and is thus taken as a reference for the test. Verification of the compatibility of the two samples can be done using machine learning techniques. More specifically, clustering techniques can be used to recognize internal structures within the data sample, and then to subsequently group the output configurations of the data sample according to such internal structures. Notably, the clustering structures which are identified depend on the class of input states (indistinguishable or distinguishable particles). The method shown in Ref. [Agre19] then works as follows. The reference sample is analysed via a clustering technique to identify its internal patterns. Then, the clustering structure obtained from this analysis is applied to both the trusted sample and the one to be tested. Statistical tests are then used to check whether these two samples can be considered compatible.

As part of the PHOQUSING project, we have then analysed the possibility to extend this approach to a more general scenario. More specifically, we first observe that pattern recognition techniques can be used in an adversarial sense. Given a sample A generated from a device that needs to be tested, one could consider generating data samples from different mock-up distributions to exclude their compatibility with the unknown sample A. For instance, one could consider generating data samples produced by a linear interferometer described by the same unitary matrix U, with input states composed of partially distinguishable photons. For certain ranges of the indistinguishability parameter x, efficient classical algorithms to produce such samples can be employed [Rene18]. These algorithms can be thus used to efficiently generate different reference samples from classically-simulable mock-up distributions.

We have thus investigated within PHOQUSING whether clustering techniques can be effectively used as a tool to validate against such classically-simulable mock-up models. Numerical simulations suggest that, by progressively increasing the number of events in the data samples, the clustering-based test becomes more stringent in assessing the compatibility of an unknown sample with respect to the reference ones with given indistinguishability parameter x. Furthermore, numerical simulations also suggest a favourable scaling of the number of required events with respect to the system size (number of photons n and m).

On one side, we observe that the adoption of clustering techniques in such an adversarial sense requires a precise characterization of the unitary matrix performed by the device. Indeed, reference samples generated by noisy unitaries, different from the one associated to the sample to be tested, are identified as incompatible by the clustering-based test. However, this is also an advantage of such an approach, as imperfections in the unitary transformation fail to generate false positive results.

2.6 Semi-device independent methods

A desirable feature in multiphoton interference certification is to make it as independent as possible from the correct control of the measurement apparatus which is performing the certification. This prevents that errors in implementing the desired measurement translate into falsely certifying indistinguishability when none is present. Given that the control theory of large-scale photonic experiments is still something that is in development, this is a problem of more than theoretical interest.

For the original Hong-Ou-Mandel effect, the security against poor implementations of the measurement apparatus comes 'for free': every deviation from the 50/50 beam splitter will reduce the observed two-photon interference, meaning that if the beam splitter is poorly set, one can fail to certify indistinguishability, but one can never falsely certify indistinguishability when none is present. However, for multiphoton interference experiments, this is not the case. In particular, for direct multiphoton generalizations of the HOM effect, it is possible to observe the same suppression laws with a different interferometer, thereby falsely certifying indistinguishability [Shch16].

One can cast this entire discussion into the language of entanglement witnesses by observing that for a generic linear optical circuit, photonic indistinguishability and multiphoton entanglement are equivalent. This allows one to import the language of device independence into the problem: if one finds an observable where there is a difference in expected value between quantum and classical particles, and moreover, where this difference persists when both quantities are independently maximized over all possible settings of some part of the measurement apparatus, then the resulting witness is independent from the measurement settings of that part of the apparatus. To emphasize that we are considering parts of the apparatus and not the whole, we use 'semi-device independent' for this effect, in agreement with earlier literature.

In an experiment using a quantum photonic processor [Meer21], we have demonstrated such a semi-device independent indistinguishability witness. The chosen observable is the two-mode correlator between a pair of optical modes [Wal16]. We find the settings of the photonic processor which maximize this quantity and show that we can exceed it using quantum resources, thereby demonstrating the semi-device independence of this witness as outlined above.

Experimentally, we certify indistinguishability in a 4-mode system into which 3 photons are injected. We certify indistinguishability between every pair of photons in our experiment, which implies three-photon indistinguishability via a triangle inequality argument.

3 Discussion and conclusion

In this report we have reviewed the methods PHOQUSING researchers are using to certify multi-photon indistinguishability, and its uses, in the devices we are developing. These methods vary in applicability, generality, and on the strength and type of assumptions that the user can, or may want to, make. Here we offer some general remarks and comparisons:

- The most limited methods are those which certify multiphoton indistinguishability based on the interference that happens only in interferometers that have certain symmetries. For example, they rely on identifying pairs of input/output states that have an associated zero amplitude, in e.g. Fourier and Sylvester interferometers. As such, they offer no direct guarantees of indistinguishability for other interferometers. On the positive side, such fixed interferometers may be used as a special device to certify multi-photon sources.
- Building on this idea, the concept of genuine multiphoton indistinguishability was proposed, and different interferometer designs to certify it were suggested and demonstrated [Brod19, Pont22]. There is on-going work in the PHOQUSING project related to finding better interferometer designs for this task.
- An independent theoretical idea developed within this project saw the identification of collective photonic phases as a photonic embodiment of a more general quantum-mechanical phenomenon, namely the invariance of some specific multi-state quantities under unitaries (the so-called Bargmann invariants) [Oszm21]. PHOQUSING is also investigating how to push the characterization of multi-photon indistinguishability in terms of these invariants.
- In many situations, we want to certify the presence of multiphoton indistinguishability in particular interferometers, which may not have the symmetries of those discussed thus far. This approach has led to the adaptation of machine learning techniques for identification of multi-photon interfererence [Agre19], as well as very flexible and efficient Bayesian inference methods [Vigg18b]. PHOQUSING is working to improve on these techniques, which have shown promise in identifying these traits in various interferometer designs.
- A more recent development is the certification of multiphoton indistinguishability based on weaker assumptions, for example that the interferometer effectively implemented is close to the ideal one [Meer21]. This approach relies on identifying suitable quantities that, when extremized over all possible interferometer designs, are naturally limited to some range, for the case of limited photonic indistinguishability. Experimental data in violation of that range thus certifies the presence of multi-photon indistinguishability, and can be used also to

quantify it, in a semi-device independent way. This type of certification may be helpful in protocols where one of more of the parties do not trust each other. This suggests there could be cryptographic applications of this kind of certification, something we aim to investigate as the project develops.

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