Deliverable Report

Deliverable No: D2.3

Deliverable Title: Robustness and tolerance to fabrication errors

Grant Agreement number: 899544

Project acronym: PHOQUSING

Project title: PHOtonic Quantum SamplING machine

Project website address: www.phoqusing.eu

Name, title and organisation of the scientific representative of deliverable's lead beneficiary (task leader):
Prof. Ernesto F. Galvão, LIN INL

Deliverable table

Deliverable no.	D2.3
Deliverable name	Robustness and tolerance to fabrication errors
WP no.	2
Lead beneficiary	LIN INL
Type	Report
Dissemination level	Public
Delivery date from Annex I	M24
Actual delivery date	30 August 2022

What was planned *(from Annex I:)*

D2.3 -- Robustness and tolerance to fabrication errors (M24);
Description: The sampling protocols we will be implementing will be realized on the noisy, intermediate scale (NISQ) devices we will build. Theoretical support for these applications will be provided by simulations and analytical studies to characterize the noise tolerances that are required for different applications. The errors arise from photon loss, limited fabrication accuracy, and limited photon indistinguishability, so we will need to investigate how these different sources of error impact the sampling tasks.

What was done.

1. Introduction

Project PHOQUSING aims to develop sophisticated multimode interferometers capable of various sampling tasks with applications in quantum computation, benchmarking of single-photon sources, and randomness manipulation, among others. This will involve building, modelling and testing photonic quantum computers based on two different physical platforms, using tunable multimode interferometers built either out of femtosecond laser-written silica (the QOLOSSUS sampling machine), or silicon nitride (the QALCULUS machine). These two platforms have different but complementary capabilities, and testing applications on both serves as a cross-validation of techniques for characterization, modelling and benchmarking.

In this report we will review some of the modelling techniques that the PHOQUSING project is using to characterize real-world, noisy devices we are developing, in particular with respect to: limited fabrication accuracies, limited photonic indistinguishability, cross-talk between optical elements, and photon loss. As a general rule, the imperfections can be operationally characterized by some distance metric between the ideal and real distributions of photons at the output – in the literature, the total variation distance is often used. Depending on the application, however, it is possible that higher levels of noise may be tolerated, or that a more coarse-grained characterization of the output distribution is sufficient, for example binning all events corresponding to a certain property being investigated (e.g. binning together all events in which bosonic bunching was observed). This motivates a review of the noise characterization that is adequate for each particular task we are exploring in the project. In the following, we will review noise-modelling techniques used in the following applications:

- Genuine multiphoton indistinguishability test based on cyclic circuits;
- Boson sampling using a 3D, continuously coupled femtosecond laser written waveguide array;
- Cross-talk characterization in large silicon-nitride tunable interferometers;
- Characterization and applications of collective photonic phases (also known as Bargmann invariants).

Taken together, these techniques provide complementary ways to characterize noise and experimental imperfections in real photonic devices, as necessary for the on-going development under way in the PHOQUSING project. Let us now review particularities of noise modelling for each of these applications.

2. Genuine multiphoton indistinguishability characterization based on cyclic interferometers

In ref. [Pont22], researchers from the PHOQUSING nodes Sorbonne, CNR, and Rome proposed an interferometer design that is capable of measuring genuine multiphoton indistinguishability, a concept put forth previously in ref. [Brod19]. The basic idea is to model a source that outputs single photon states that can be either perfectly identical, or mutually orthogonal. The density matrix describing states prepared by this source is of the form:

$$\rho = c_1 \rho^{\parallel} + \sum_i c_i \rho_i^{\perp}$$

Where $\rho^{\parallel}$ is a state with indistinguishable photons, and $\rho^{\perp}$ are states for which at least one pair of photons are in mutually orthogonal (distinguishable) states. The coefficient c_1 was shown to be a good characterization of the extent to which the multiple photons are mutually indistinguishable.

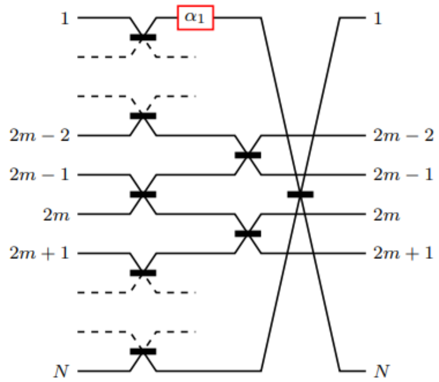

Figure 1. Interferometer design with two layers of beam-splitters, proposed in [Pont22] for estimating the coefficient of genuine multiphoton indistinguishability from the visibility obtained when varying the phase α_1, using preparation and detection of single photons in odd-numbered inputs and outputs.

The interferometer design proposed in [Pont22] to characterize c_1 for any number of photons is shown in Fig. 1. It consists of two layers of beam splitters (BSs), with a single BS nonlocal connection between the top and last BS of the second layer. The interferometer is intended to be used with n single-photon states, each entering the interferometer in one of the odd-numbered input modes. Photo-detection at the output is done by postselecting only events in which photons are detected at the same set of modes used for the input photons (all odd-numbered modes). It was shown that the probability of such events is given by:

$$P' = \frac{1}{2^{2n-1}}[1 + (-1)^n c_1 \cos \alpha_1].$$

This means that by varying the single phase alpha, we expect to see an oscillation of this probability, whose visibility is given by c_1.

The modelling of noise and experimental imperfections, detailed in the Appendix of [Pont22], includes four main components:

- *Multiphoton emission and losses*. This was the main source of errors, corresponding to a finite probability $g^2(0)$ that the source emits more than one photon per mode. It can be shown that, for quantum-dot sources, the unwanted (noise) photon will be orthogonal to the signal photons, and the probability associated with this noise photon can be estimated from the source's brightness. For the estimation of losses, the symmetry of the different arms of this interferometer was used, which allows an estimate of equal losses in all arms. With this assumption, it is possible to use the results of [Oszm18], and the losses could be modelled as happening at the input of the 8-mode interferometer, via a single η transmission parameter per photon.
- *Partial photon indistinguishability*. While in previous works on boson sampling experiments [Tich15, Rene18], modelling via general Gram matrix of inner products between single-photon spectral functions was used, this particular experiment was motivated by an assumption about the input states (associated with the concept of genuine multiphoton indistinguishability), so each photon was modelled as having a probability x_i to be indistinguishable from the others, and $(1 - x_i)$ to be perfectly distinguishable from the others. This corresponds to a simplified Gram matrix, which nevertheless found good

agreement with the experimental data reported. The parameters x_i for each photon were obtained from the Hong-Ou-Mandel visibilities of the pairs, some of which could be observed in the same 4-photon set-up, by estimating the bunching probabilities at particular pairs of output modes.

- *Imperfections in the circuit parameters*. The simplified interferometer design was helpful for the characterization of circuit parameters, as only 50/50 beam-splitters are required, with an ideal transmissivity $T=0.5$, and which were measured in independent experiments prior to the 4-photon run. The experimentally characterized T were all within 3.4% of the ideal value of 50% transmissivity.
- *Unbalanced detection efficiencies*. Detectors will typically have different detection efficiencies. These can be characterized individually in independent experiments, and the imbalances were taken into account to correct the observed event probabilities in the 4-photon experiment.

Taking all the imperfections into account, in Fig. 2 we can see how the experimental data (black dots) compare with the simulation that includes imperfections (orange), and how unwanted multiphoton emissions decrease the value of the genuine indistinguishability parameter c_1 with respect to the case without multiphoton emissions (green). This was plotted as a function of the pairwise indistinguishability.

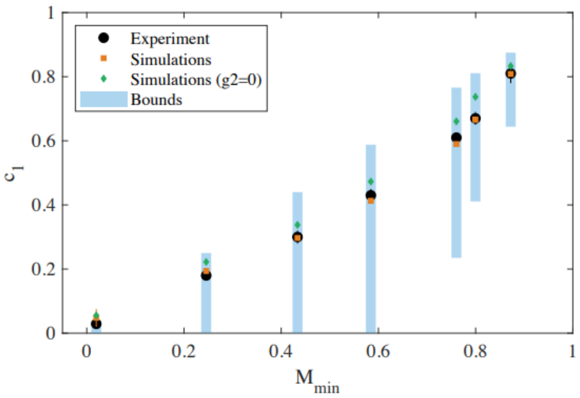

Figure 2 Simulation including various sources of noise, and experimental data for the genuine indistinguishability coefficient c1, as a function of pairwise indistinguishability M_{min}. Taken from [Pont22].

3. Boson sampling in continuously coupled 3D waveguides in silica

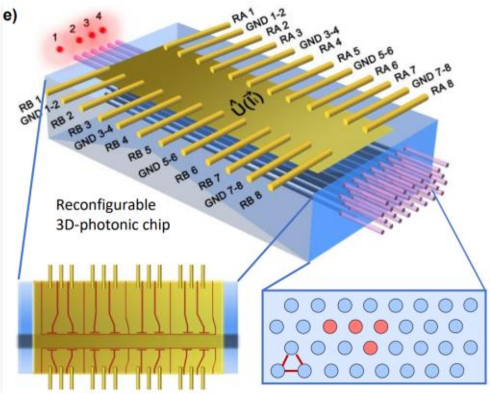

Figure 3 . 3D laser-written multimode interferometer demonstrated in boson sampling experiments reported in [Hoch22]. 16 resistors on the device's surface can be controlled to locally change the index of refraction, effectively reconfiguring this 32-mode device. Waveguides are coupled by evanescent field coupling in a triangular lattice (bottom right).

In the recent paper [Hoch22], PHOQUSING partners in Sapienza/Rome and CNR/Milan reported device characterization and boson sampling experiments using a tunable interferometer with 32 continuously-coupled waveguides. The waveguides are laser-written in 3D in a glass substrate, and can be tuned by using 16 resistors on the device's surface. In common with other quantum computational advantage experiments, boson sampling tasks involve sampling from the distribution of photons at the output of a randomly chosen device. This task has some guarantees of hardness, and randomness plays a role by preventing that symmetries simplify the simulation for the classical computer.

The first focus of this work is on device characterization, to show the capability of reconfiguration, as well as the required measurements for characterizing the device. The device is reconfigured using the surface resistors, and the interferometer implemented characterized via the measurement of 496 Hong-Ou-Mandel dips for each input pair, which is a sign of the complexity of experimentally reconstructing such general linear-optical dynamics. The statistics for phase and absolute value of the unitary that describe the interferometers being implemented was compared with uniformly drawn unitaries, showing signatures of randomness. As randomness is a desired feature for quantum advantage experiments using boson sampling, for this application it is important to characterize how well the interferometers obtained approximate elements of the uniformly random Haar ensemble. In Fig. 3 we see a histogram showing the similarity of different columns of the interferometer unitary, both for the case of experimentally-obtained random configurations of the thermos-optical couplers, and for the mathematical ideal case of uniformly-drawn interferometers. The distributions are not too dissimilar, showing some quantitative evidence of the randomness attained by the reconfigurable devices.

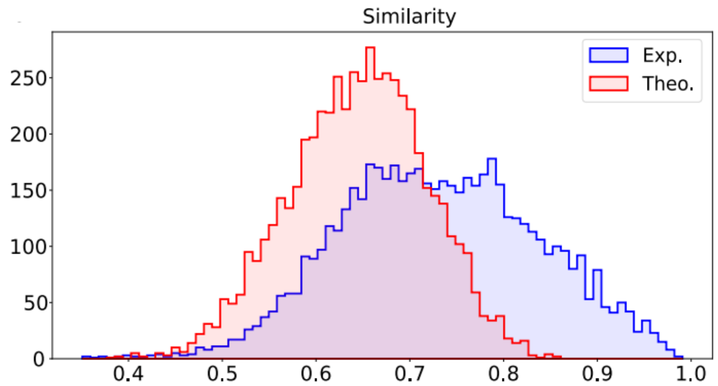

Figure 4 Distance metric (similarity [Hoch22]) between output data obtained with various random settings of thermo-optical couplers, and ideal-case interferometers obtained from sampling the uniform, Haar ensemble. The x-axis represent the similarity between different columns of the same random interferometer.

Besides device characterization, boson sampling experiments were performed using three and four input photons. In this type of application using large-scale devices, it is practically impossible to fully characterize the device, due to the large number of possible outputs, and the sampling complexity associated with characterizing each with the relative frequencies of output events. Instead, a Bayesian approach has been often used, and this is the approach taken in this work. Each sampled event has its probability calculated according to the hypothesis that the device is working as a perfect sampler, but also according to different hypotheses describing noise. The first alternative hypothesis considered is that of complete, white noise, where the interferometer samples from a uniform distribution at the output. A second hypothesis is that of complete loss of photonic indistinguishability, but where the outputs are still sensitive to the dynamics, only via the dynamics of distinguishable photons. Bayesian hypothesis testing was done using the experimental data, showing strong evidence that the devices are outputting outcomes that are closer to perfect samplers than they are to being described by these two noise models (see Fig. 4).

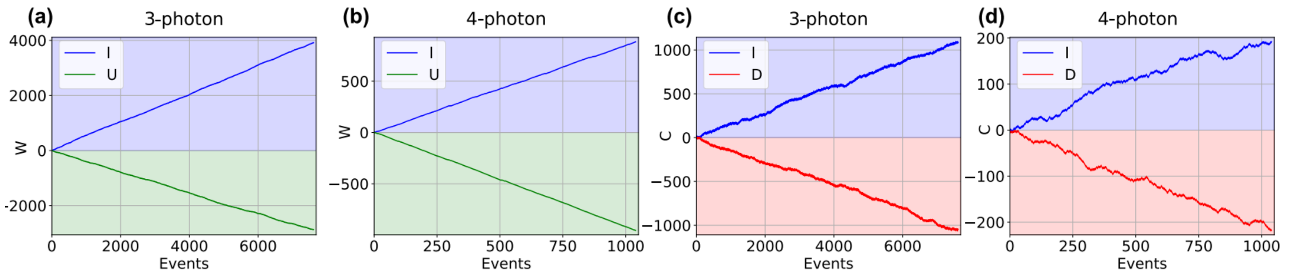

Figure 5 Bayesian validation of experimental boson sampling data of 3- and 4-photon experiments, against the hypotheses that events are uniform (a, b), or that photons are distinguishable (c, d). The graph shows a counter that increases for each event identified as closer to the ideal boson sampling distribution, and decreases for events identified as closer to the alternative noise model. Taken from [Hoch22].

4. Cross-talk characterization in large, reconfigurable silicon-nitride interferometers

The implementation of unitary transformations with high fidelity, i.e. low noise, on a quantum photonic processor (QPP) requires accurate control of the phase shifts on the processor. In the case of the QPPs developed as part of the PHOQUSING project, phase shifts are applied into the integrated circuits by placing resistive heaters on-top of the waveguide. Here, the phase shifts are induced by applying voltages to the heaters, changing the refractive index on specific target waveguides.

This results in two major sources of noise that lower the fidelity of the programmed processor if they are not corrected for, namely thermal and electrical crosstalk. Thermal crosstalk is when the generated heat to achieve a specific phase change spreads out far beyond the target waveguides, which results in phase shifts at unintended locations. An illustration of such a spread in a QPP is shown in figure 6. Here, a phase shifter, ϕ_h, induces an undesired phase shift on neighbouring waveguides of a QPP (here a rectangular interferometric mesh) as used in the Calculus machine. Thermal crosstalk is a major source of errors in heater-based quantum photonic processors. These unintended phase shifts will lower the fidelity of any implemented unitary transformation. There are two ways of combating thermal crosstalk: first, reducing its effect by changing the physical design parameters of the QPP, e.g., by increasing the distance between waveguides, second, correcting the phase settings for the thermal crosstalk so that the corrected phases can better implement the desired unitary transformation [DiGi20]. PHOQUSING partner QUIX has developed software to dial arbitrary settings in tunable interferometers, using the measurement results to correct for thermal cross-talk between couplers. A preliminary application of this computational approach to noise mitigation raised the average coupler fidelity from $F = 0.743 \pm 0.049$ to $F = 0.992 \pm 0.002$.

On the other hand, electrical crosstalk is a result of a non-zero resistance of the on-chip electrical leads which connect the resistive heaters with the drivers used to apply the voltage. The non-zero resistance results in a voltage drop over the electrical lines and hence an incorrect setting of the optical phases, which also has an influence on the resistive heater sharing the same ground. This effect can be taken care of by careful consideration of the on-chip circuitry [Prab20].

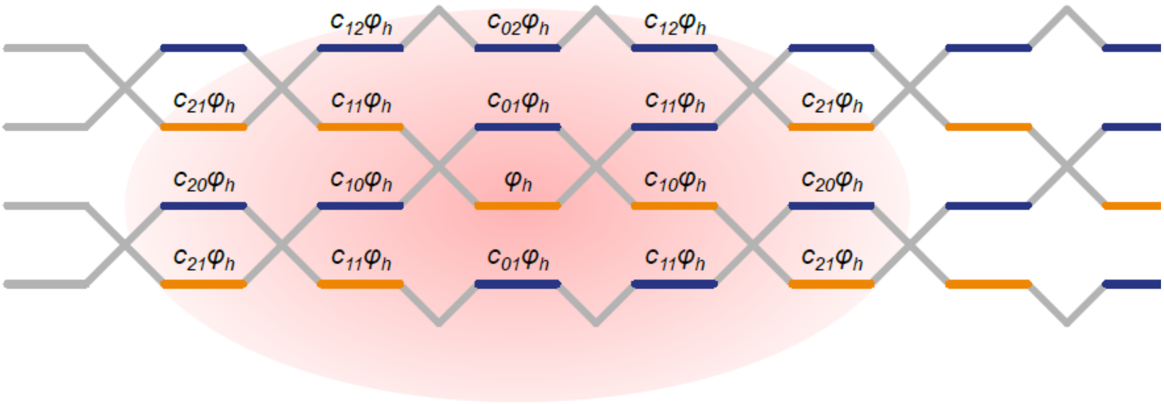

Figure 6 Illustration of thermal cross-talk on neighbouring waveguide sections in a quantum photonic processor (QPP).

5. Characterization and applications of collective photonic phases

A number of experiments have experimentally measured so-called collective photonic phases [Mens17, Jone20]. These are complex-valued phases that arise in a variety of settings, in particular in the characterization of multi-photon indistinguishability. In a number of recent experiments, these phases were measured for 3 or 4 photons, as an example of a curious phenomenon that is genuinely multi-system, as it can only be probed when you have a minimum number of single-photon states. In a recent preprint supported by the PHOQUSING project, the INL node and collaborators gave a solid foundation for the meaning and uses of these phases in quantum mechanics in general. In [Oszm21], it was shown that these are the phases of so-called Bargmann unitary invariants, which have the following form:

$$\Delta_{12\ldots n} = Tr(\rho_1 \rho_2 \ldots \rho_n),$$

Where $\rho_1, \rho_2, \ldots, \rho_n$ are single-system states. Note that we have a product of inner products over a cycle of quantum states, and the cycle structure guarantees that these quantities remain invariant if all states undergo the same unitary evolution. This means that these quantities are physical, can be measured, and pertain to the relational information having to do with the geometrical arrangement of the states in the cycle. In [Oszm21], a general circuit for measuring these unitary invariants was proposed, together with a way to combine the information of a complete set of these invariants to completely characterize the geometrical arrangement of the states. This enables applications such as dimension and coherence witnesses, as well as imaginarity witnesses, i.e. circuits that can identify the presence of complex-valued amplitudes in the set of input states. Measuring such quantities is a more direct and efficient way to enable these applications; instead of performing full state tomography for each state in the tuple, it is sufficient to measure a (potentially exponentially smaller) number of invariant parameters directly, from which all the required information can be obtained in a classical post-processing step.

While the preprint [Oszm21] deals with the problem in an abstract setting, describing how to measure and use these invariants in a platform-agnostic way, the effect of noise was also considered. This is a first step towards an analysis of how the noise may affect applications based on the measurements of these invariants. In particular, some robustness results were obtained, showing how states with limited purity can still be characterized in this way, which is an experimentally relevant approximation that applies to possible experimental photonic implementations within the PHOQUSING project.

Preliminary results indicate that the cyclic interferometers of PHOQUSING preprint [Pont22] may also be used to measure these invariants; PHOQUSING is looking into the most efficient ways of measuring these invariants in a photonic setting, for some of the applications outlined in this first theoretical preprint [Oszm21].

6. Conclusion

Any quantum circuit implementation with current technologies will be plagued by experimental noise. In this Noisy, Intermediate-Scale Quantum (NISQ) device era, it is extremely important to known how to characterize the noise in real devices, and their effect in different applications. The current activities of project PHOQUSING, reviewed here, explored the effect of noise in our two physical platforms in various ways:

- We know the linear-optical evolution of distinguishable photons can be efficiently simulated on a classical computer. It is thus essential for any genuinely quantum application to rely on photonic indistinguishability. In [Pont22], we modelled losses, unwanted multiphoton emissions, limited indistinguishability, and manufacturing uncertainties in a comprehensive study of the effect of noise in a certification of genuine multi-photon indistinguishability using a cyclic interferometer in the platform used to develop our QOLOSSUS machine.
- Another experiment with the QOLOSSUS platform implemented a 3D boson sampling machine [Hoch22]. For this application of quantum computational advantage, it is important to guarantee there are no regularities in the random interferometers chosen. We have, accordingly, performed statistical tests on the randomness achieved, as well as Bayesian tests of photonic indistinguishability, of the kind suitable to larger interferometers for which a single amplitude still can be calculated in reasonable time on a classical computer.
- We have identified cross-talk as one of the main sources of error in dialling arbitrary interferometer designs in QALQULUS, our platform capable of reaching the largest number of modes. We have analysed this cross-talk, and developed software to mitigate these errors.
- In [Oszm21] we have also studied, from a more theoretical perspective, how collective photonic phases are an example of a more general quantum-mechanical phenomenon, associated with the existence of multi-state quantities that are invariant under unitaries. We have also given the first steps towards error-tolerance, studying robustness of the characterization using mixed-state versions of the (ideally pure) states.

7. Bibliography

[Brod19] D. J. Brod, E. F. Galvão, N. Viggianiello, F. Flamini, N. Spagnolo, F. Sciarrino, “Witnessing genuine multiphoton indistinguishability”, Phys. Rev. Lett. 122, 063602 (2019).

[DiGi20] N. Di Giano, Master thesis, Politecnico Milano, (2020) https://www.politesi.polimi.it/handle/10589/185821

[Hoch22] F. Hoch, S. Piacentini, T. Giordani, Z.-N Tian, M. Iuliano, C. Esposito, A. Camillini, G. Carvacho, F. Ceccarelli, N. Spagnolo, A. Crespi, F. Sciarrino, R. Osellame, “Reconfigurable continuously-coupled 3D photonic circuit for Boson Sampling experiments”, npj uantum Inf 8, 55 (2022).

[Jone20] A. E. Jones,, A. J. Menssen, H. M. Chrzanowski, T. A. W. Wolterink, V. S. Shchesnovich, and I. A. Walmsley, “Multiparticle Interference of Pairwise Distinguishable Photons”, Phys. Rev. Lett. 125, 123603 (2020).

[Mens17] A. J. Menssen, A. E. Jones, B. J. Metcalf, M. C. Tichy, S. Barz, W. S. Kolthammer, and I. A. Walmsley, “Distinguishability and Many-particle Interference”, Phys. Rev. Lett. 118, 153603 (2017).

[Oszm18] M. Oszmaniec and D. J. Brod, “Classical simulation of photonic linear optics with lost particles”, New J. Phys. 20, 092002 (2018).

[Oszm21] M. Oszmaniec, D. J. Brod, E. F. Galvão, “Measuring relational information between quantum states, and applications”, preprint arXiv:2109.10006 [quant-ph].

[Pont22] M. Pont et al., “Quantifying n-photon indistinguishability with a cyclic integrated interferometer”. arXiv preprint arXiv:2201.13333 [quant-ph].

[Prab20] M. Prabhu et al., “Accelerating recurrent Ising machines in photonic integrated circuits”, Optica 7, 551-558 (2020).

[Rene18] J. J. Renema, A. Menssen, W. R. Clements, G. Triginer, W. S. Kolthammer, and I. A. Walmsley, “Efficient classical algorithm for boson sampling with partially distinguishable photons”, Phys. Rev. Lett. 120, 220502 (2018).

[Tich15] M. C. Tichy, “Sampling of partially distinguishable bosons and the relation to the multidimensional permanent”, Phys. Rev. A 91, 022316 (2015).