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Deliverable table

Deliverable no.	D1.1
Deliverable name	Classification of sampling tasks with identification of estimators
WP no.	1
Lead beneficiary	LIN INL
Type	Report
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Delivery date from Annex I	M18
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What was planned *(from Annex I:)*

D1.1– Classification of sampling tasks with identification of estimators (M18);

Report on the classification of variations of sampling tasks achievable with linear optics, and equivalences between them. These include Boson Sampling (BS), Scattershot BS, Driven BS, Gaussian BS, non-local BS. New schemes will also be described, such as sampling with inputs consisting of photon number superpositions. We will characterize the enhancement provided by nonlinearities at the few-photon level via the postselection overhead induced by a linear-optical simulation of the nonlinear process.

What was done.

1 Introduction

In this report we review variations of sampling tasks achievable with linear optics, and equivalences between them. These equivalences may be in terms of simulation effort; experimental resources required; and trade-offs possible with different architectural choices. The variations we review are:

- Fock-state Boson Sampling (BS);
- Scattershot BS (SBS);
- Driven BS (DBS);
- Non-local BS (NLBS);
- Constant-depth BS (CDBS)
- Gaussian BS (GBS);
- Bipartite GBS;
- BS using superpositions of Fock-state inputs;
- Non-linear BS.

This report is structured as follows. In section 2 we briefly review the main characteristics of each of these variations. In section 3 we highlight equivalences between the different models, whereas in section 4 we have some general remarks regarding the relevance of the different models to the goals of PHOQUSING.

2 Brief survey of variations of photonic sampling models

2.1 Fock-state Boson Sampling (BS)

Fock-state Boson Sampling was originally proposed by Aaronson and Arkhipov in [Aaro11] as a way to showcase quantum computational advantage using Fock-state inputs of n single photons, evolution via m-mode linear interferometers described by (Haar) uniformly random mxm matrices, with photo-detection at the output. By choosing $m=O(n^2)$ we can avoid collisions, i.e. more than one photon per output mode [Arkh12]. With collisions suppressed, an experimental implementation can be done with bucket detections, which do not discriminate photon numbers. Having $m>>n$ is also required to guarantee that the experimental outcomes will have probability amplitudes that are proportional to the permanent of matrices that are approximately Gaussian, and hence do not have any structure that could decrease the classical simulation complexity. The permanent function is notoriously hard to compute, and its exact calculation is known to be in computational complexity #P.

The hardness of classical simulation depends on three technical, complexity-theoretical conjectures related to anti-concentration of permanents of Gaussian matrices, avoidance of the collapse of the polynomial hierarchy of computational complexity, and that permanents of Gaussian matrices are hard to approximate. There have been several experimental implementations of BS, with the current record being an experiment with quantum dot sources of single photons, where 20 photons were sent in the device, with up to 14 photons detected at the output [Wang19].

2.2 Scattershot Boson Sampling (SBS) and driven Boson Sampling (DBS)

Scattershot Boson Sampling (SBS) [Hami17, Krus19] was proposed as a variation of BS that takes advantage of a randomized setting to increase the event rate of a BS set-up using probabilistic single-photon sources, in particular parametric down-conversion (PDC) sources (see Fig. 1). These sources use a parametric down-conversion crystal that probabilistic generate twin photons from single photons in the pump beam. When successful, the detection of the idler photon serve to herald the presence of the signal photon, which goes in the interferometer in a known input mode. The key idea of SBS is to use multiple such heralded PDC sources, with up to one per input mode of the interferometer. The heralded input photons will enter various combinations of input modes, but the fact that we can use *m>>n* sources to postselect events with n input photons greatly enhances the event rate.

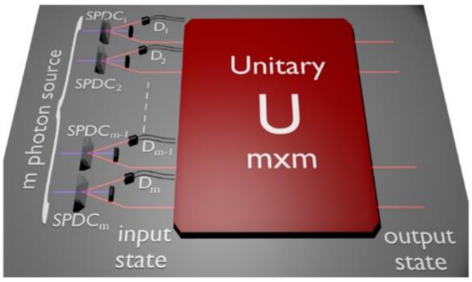

Figure 1 Experimental scheme for Scattershot Boson Sampling using multiple parametric down-conversion sources. Taken from [Bent15], with permission from the authors.

Another particularity of this scheme, which we will return to in section 4, is the fact that here we are using Gaussian state sources (the PDC crystals) to create a heralded source of single-photon states. This is only approximate, given that the PDC source also generates higher-order events corresponding to more than one pair of photons, which for the usual source set-ups will go unnoticed, and pass for a single pair.

Driven Boson Sampling (DBS) [Bark17] is a variation of SBS, where more than *m* single photons can be inputted to the *m*-mode, Haar-random interferometer. This is achieved by using a state-preparation, linear-optical stage *G* consisting of *km* modes, where multiple photon addition operations can be performed. This effectively couples *km* heralded single-photon sources to the Haar-random, *m*-mode interferometer H where we will perform a variation of an SBS experiment. The result is a *k*-fold increase in the event rate, or alternatively, a *k*-fold decrease in the pump power required for a fixed number of photons at the input. Decreasing pump power is useful, as fewer higher-order events with more than 2 photons will be generated, thus improving the quality of the PDC single-photon sources.

2.3 Non-local Boson Sampling (Non-local BS)

As we have briefly discussed when reviewing SBS, the key idea is to improve event rate by having up to one SPDC source coupled to each input mode of the interferometer. Let us consider each SPDC source – it consists of an input of the pump beam, and two output modes. One output is sent to the interferometer (which is in Alice's lab), while the second output (where the idler photon goes) is coupled to a photodetector, whose signals herald the presence of a photon at Alice. One way of viewing this set-up is to imagine that all modes where the idler photons can be are sent to Bob's laboratory. We can they analyse this SBS scheme from the point of view of the non-local correlations that arise between Alice and Bob – this was called the Non-local Boson Sampling model in [Shah17]. For example, it can be shown that if no information from Bob is used, the state at Alice is effectively a classical thermal state with non-negative *P* quasi-probability distribution. Thus, while the set-up is not different from SBS, looking at the experiment from this bipartite point of view leads to some interesting observation about the correlations that arise, for example the thermal nature of the reduced density matrices. Perhaps more importantly, this paper was a precursor to GBS, where the Gaussian nature of the inputs moves from a peculiarity of SPDC sources, to an intrinsic feature that can be exploited computationally, as we have briefly discussed in the report associated with PHOQUSING Deliverable 7.1 "Development and classification of HQC based on non-adaptive linear optics".

2.4 Constant-depth Boson Sampling (CDBS)

In the original Boson Sampling proposal by Aaronson and Arkhipov [Aaro11], the requirement that the interferometers' unitaries be uniformly drawn from the Haar ensemble requires, in principle, a universal interferometer design. Assuming local connectivity between the optical elements, as in the designs proposed by Clements *et al.* [Clem16], universal m-mode interferometers require $O(m)$ layers of beam-splitters, or equivalent optical elements. Reference [Brod15] analysed the complexity of constant-depth Boson Sampling (CDBS), that is, Fock-state BS using interferometers having just a few layers of beam-splitters. Using a correspondence with constant-depth circuits, and assuming arbitrary connectivity between the layers, [Brod15] showed that BS experiments using 4 or more layers are hard to simulate exactly; that CDBS with one or two arbitrarily connected BS layers can be simulated exactly efficiently (in the weak sense, i.e. sampling), and left open the case of interferometers with 3 layers. These results, however, did not address the experimentally important case of locally connected designs, such as the commonly used one by Clements *et al.* [Clem16] (see Fig. 2).

As the experimental amplitudes of Fock-state BS are given in terms of permanents, any algorithm capable of calculating the permanent of the relevant submatrices of the interferometer's unitary U can be used for strong simulation, i.e. amplitude calculation. We have identified two approaches that can speed up permanent calculation for interferometers with a constant number d of locally-connected beam-splitter layers. In the Clements *et al.* design [Clem16], after d layers of beam-splitters an input mode is connected to at most $2d$ neighboring output modes (see Fig. 2). This means the unitary matrix U describing the interferometer is a band matrix, having non-zero elements only in a band of bandwidth of size $O(d)$ around the main diagonal. An experiment with a single photon per output and input mode corresponds to calculating the permanent of U, and for band matrices, Cifuentes and Parrilo [Cifu16] have described an algorithm whose time complexity is linear in the matrix size m, while being exponential in the bandwidth. For CDBS outcomes other than having a single photon per mode, we need to compute a permanent of a submatrix of U with repeated columns, which enlarges the band width, and hence the computation time.

An alternative approach to simulating constant-depth BS with local beam-splitter connectivity is to use a Feynman path sum approach, summing the amplitudes to all possible intermediate waveguide occupation numbers. Preliminary results indicate that whereas the memory and time required increase exponentially with the depth d, the simulation is efficient with respect to the number of modes m.

PHOQUSING investigates new techniques for the simulation of medium-scale linear-optical devices, and using the Cifuentes-Parrilo algorithm, as well as Feynman path sums, are among the innovations developed in the project.

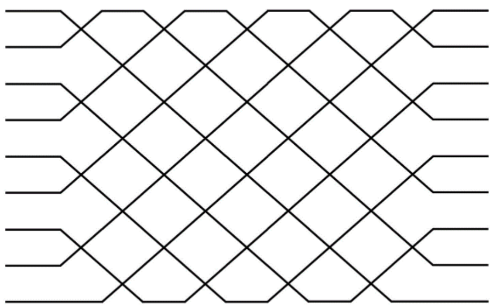

Figure 2 Illustration of the interferometer design proposed in [Clem16]. Lines represent waveguides, and line crossings represent beam-splitters. The figure shows a universal design for 9-mode interferometers.

2.4 Gaussian Boson Sampling (GBS)

Gaussian Boson Sampling [Hami17, Krus19] is a photonic computation model that inherits the linear optical evolution of Boson Sampling, as well as the photodetection at the end (see Fig. 3). The difference with respect to Boson Sampling is in the inputs, which are now Gaussian states. It is known that Gaussian state evolve in linear optics in a way that can be efficiently simulated classically, using a phase-space representation. If no non-Gaussian operation was available, the whole computational process would be efficiently simulable on a classical computer [Bart02, Mari12] – for example, homodyne detection is a Gaussian operation that allows for efficient classical simulation. The non-Gaussian ingredient introduced in this model is the photodetection at the output, which puts exact GBS in the same footing, from the point of view of computational complexity theory, as Fock-state Boson Sampling. This is reflected by the fact that the GBS probability amplitudes are proportional to the hafnian of a matrix associated with the interferemeter design and the choice of squeezing parameters for the input states – evaluating the hafnian is in the same computational complexity class as evaluating the permanent. More precisely, the probabilities are given by:

$$\Pr(S) = \frac{1}{\mathcal{Z}} \frac{|\mathrm{Haf}(A_S)|^2}{s_1!s_2!\cdots s_{2m}!}$$

$$\begin{aligned} \mathcal{Z} &= \prod_{i=1}^{2m} \cosh(r_i), \\ A &= U\,\mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_{2m})U^T \\ \sigma_i &= \tanh(r_i), \quad 0 \leq \sigma_i < 1 \end{aligned}$$

where r_i are the squeezing parameters, U is the unitary matrix representing the interferometer, and A_S is built from A by repeating the ith row and column of A s_i times, with s_i being the occupation number of output mode i.

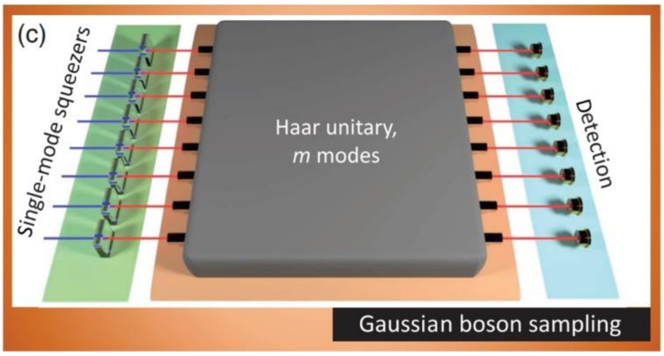

Figure 3 Gaussian Boson Sampling (GBS) set-up. Taken from [Brod19], with permission from the authors.

One advantage of GBS with respect to BS is that there is more freedom involved in the preparation of the input states, as the squeezing parameter of each single-mode input Gaussian state can be used to encode information. The Autonne-Takagi decomposition [Brom20] of an arbitrary symmetric matrix is used to choose interferometer and squeezing parameters so that experimental amplitudes are proportional to Hafnians of arbitrary symmetric matrices, encoding for example the adjacency matrix of a graph. This has been used to propose GBS applications to graph theory, optimization, and simulation of molecular vibronic spectra, some of which were reviewed in PHOQUSING's report associated with deliverable D7.1 "Development and classification of HQC based on non-adaptive linear optics".

Using squeezed vacuum inputs, impressive quantum computational advantage experiments were performed [Zhon20, Zhon21], with up to 113 photons detected at the output. This put a photonic platform as one of the only two physical platforms where convincing quantum computational advantage experiments were performed, together with experiments using superconducting chips, by the Google Quantum AI team [Arut19], and the University of Science and Technology of China [Gong21].

2.5 Bipartite Gaussian Boson Sampling (Bipartite GBS)

Bipartite Gaussian Boson Sampling is a variation of Gaussian Boson Sampling that uses multiple two-mode squeezed Gaussian states as inputs. Differently from GBS, here we have two different interferometers *U* and *V*. Each two-mode squeezed state source has one output mode sent as an input to *U*, and the other to *V* – this is the bipartition of the interferometer into two smaller ones, which does not happen in GBS (see Fig. 4).

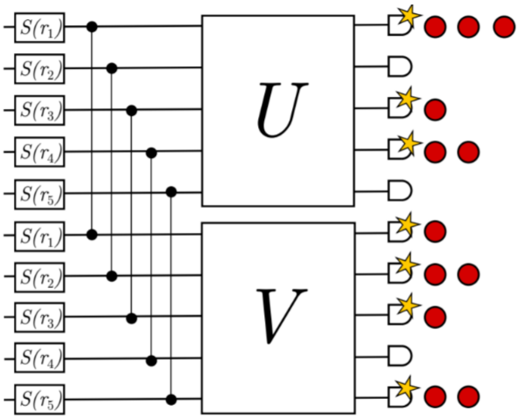

Figure 4 Experimental set-up scheme for Bipartite Gaussian Boson Sampling. Two-mode squeezed states are sent into a bipartite linear interferometer, with photodetection at the output. Figure taken from [Grie21], with permission from the authors.

There are some advantages to doing GBS in this variation. One is the fact that one can encode arbitrary complex matrices into this set-up, meaning that we can take any complex matrix *C*, and prepare *U*, *V*, and input state squeezing parameters so that the output probability amplitudes are proportional to permanents of submatrices of *C*. This makes use of *U*, *V* and the singular value decomposition of *C*, and is a more flexible way of encoding information into this kind of experiments than GBS, which can do this only for symmetric matrices. This increased flexibility in choosing the distributions that the device can sample from may lead to enhanced applications with respect to GBS. Another advantage is a proof for the hardness of exact simulation of the results, under the same assumptions of BS, but which rigorously shows hardness of simulation for $m=O(n^2)$, while for BS the only rigorous proof was for $m=O(n^5)$.

2.6 Boson Sampling using Fock-state superpositions (Superposition BS)

Superposition boson sampling is the variant of boson sampling where instead of single photon Fock states, coherent superpositions between zero and single photons are used. This model is of interest because it is a generalization of the original Fock state boson sampling proposal, single photon Fock states being a special case of a superposition between zero and one photons.

Practically, this variant is of interest because these states can be produced efficiently from quantum dot photon sources [Lore19], meaning that this protocol can be implemented directly using available photonic hardware.

Conceptually, this variant of boson sampling is interesting because it broadens the range of sampling problems that can be implemented on near-term photonic hardware. In Fock state boson sampling, the sampling is over permanents, i.e. $P_k = |Perm(M_k)|^2$, where M is a submatrix of a larger unitary matrix U formed by taking the rows of U where photons are incident, and the columns corresponding to the output configuration whose probability we wish to compute. In superposition sampling, in contrast, the probabilities are given by $P_k = \sum_{\sigma,\tau} Perm(M_\tau)Perm(M_\sigma)^\dagger$, where σ and τ are formed by taking all ways to select *k* photons out of the incident *n* sources. The protocol then proceeds as before: M_σ is formed by taking the rows corresponding to σ and the output columns of interest, and so on. It should be noted that unlike Fock state boson sampling, the presence of cross terms introduces a phase dependence of the output probability on the input quantum state.

Regarding the computational hardness of superposition boson sampling, several independent unpublished hardness proofs exist, which rely on the reduction of scattershot boson sampling to superposition sampling. It is also known that superposition boson sampling exhibits sufficiently large multiphoton interference contributions to be computationally hard [Rene20].

2.7 Non-linear Boson Sampling (NLBS)

Non-linear Boson Sampling, proposed in the PHOQUSING-supported preprint [Spag21], marks a departure from the variations analysed up to now, in that the evolution no longer corresponds to linear optics. Instead, at some points in the otherwise linear interferometer we introduce non-linear gates capable of implementing photon-photon interactions, and can either use inputs of continuous variables (Gaussian states) or discrete variables (Fock-states). In [Spag21] some preliminary results were obtained on the complexity introduced by a single non-linear gate of the simplest type, a single-mode non-linear phase gate that implements a phase $\exp(i\varphi n^2)$, where n is the mode photon occupation number.

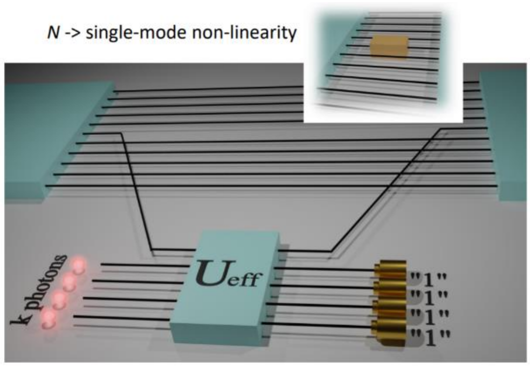

Figure 5 Experimental scheme for Non-Linear Boson Sampling (Nonlinear BS). In the inset, brown block indicated a single-mode non-linear gate. It is simulated by the introduction of a linear-optical post-selection gadget (main figure), using auxiliary photons. Figure taken from [Spag21], with permission from the authors.

To obtain insight about the complexity of this process, [Spag21] looks at a linear-optical simulation of the non-linear gate (see Fig. 4), where the single non-linear gate is introduced in the middle of a linear interferometer. The linear-optical simulation effectively implements the non-linear gate via the use of auxiliary photons inserted and detected in a second linear-optical interferometer (called the postselection gadget), and the cost can be gauged by quantifying the both the number of auxiliary photons using in the gadget, and the overhead involved in measuring all photons in the gadget, as detailed in [Spag21]. Collisions of only up to 2 photons can be expected in the asymptotic setting of $m=O(n^2)$, due to the bosonic birthday paradox that limits the collision rate as a function of the scaling of m with n [Arkh12] . This means an asymptotically exact simulation is possible with preparation and detection of only two auxiliary photons in the gadget. The postselection required means a large fraction of the linear-optical simulation experimental shots have to be discarded – this fraction ranges roughly from 85% to 90% of all events, depending on the value of the phase φ. For finite (non-asymptotic) numbers of photons and modes, linear-optical simulations of the non-linear phase gate acting on up to 4 photons were reported. In this case of experimental interest, an exact simulation of the action of the non-linear phase gate is obtained if we use as many auxiliary photons as the number of photons in the main interferometer, with limited-accuracy approximations if fewer photons are used. Note, however, that the computational problem of finding the design for the linear-optical auxiliary gadget seems to be hard, as the scheme reported relies on the evaluation of permanents of matrices of a size given by the number of auxiliary photons used.

3 Equivalences between the variations in photonic sampling tasks

In this section we overview equivalences between the different proposed variations of photonic sampling tasks. These equivalences help us identify features that unite the different proposals, and may help propose new different ways of generalizing this framework to define sampling tasks for photonic processors developed within the PHOQUSING project.

3.1 BS with SPDC sources as an example of non-local BS, GBS, and Scattershot BS

The BS model assumes sources of ideal, single-photon Fock states. Experimental work quickly found ways to approximate such sources using Spontaneous Parametric Down-Conversion (SPDC) sources and postselection. A SPDC source prepares a two-mode squeezed photonic state with higher photon number contributions that decrease monotonically as we increase the photon number. Using sufficiently low pump power, the first non-vacuum term consists of a pair of entangled photons, with the next contribution (corresponding to two pairs) having a much smaller amplitude. By detecting the idler photon of the pair we herald the presence of the signal photon, creating an approximate single-photon source. The approximation will become better as we lower the power of the pump beam, and the error incurred can be estimated, changing the output distribution in ways that can be accounted for, and indeed have been, in the multiple reported experiments using SPDC sources.

This use of SPDC sources, initially considered merely as a practical way of approximating the ideal single-photon sources, can be viewed as a particular example of GBS, where only two-mode squeezed states are used, and where one mode from each such SPDC source is coupled to a photodetector. It is also an example of non-local BS, which is just a formulation of BS with SPDC sources that draws attention to the correlations between the heralding detectors and the remaining detectors at the output of the linear-optical setup. As we have already seen, Scattershot BS uses multiple probabilistic single-photon sources to obtain a randomized-input version of BS. These probabilistic sources can be SPDC sources, and in fact that was the way the first such experiment was realized [Bent15]. We thus see that Scattershot BS experiments done via simultaneous pumping of multiple SPDC sources is also an example of GBS, relying as it does on the generation of two-mode squeezed states.

3.2 Permanents, hafnians, and the relation between GBS and Fock-state BS

As we have seen, the amplitudes of Fock-state BS are proportional to the permanents of submatrices of the unitary U that describes the linear interferometer used. GBS, on the other hand, has its amplitudes given by hafnians of matrices which incorporate both the interferometer design, and the squeezing parameters used for the Gaussian state inputs. The relationships that we have already pointed out between GBS and BS indicate that there must be a connection between permanents and hafnians. A simple connection was pointed out in the original GBS proposal [Krus19], and is actually the basis for the first hardness-of-simulation GBS result:

$$\mathrm{Perm}(G) = \mathrm{Haf}\begin{pmatrix} 0 & G \\ G^t & 0 \end{pmatrix}$$

This equation shows the Hafnian is more general than the permanent, and that approximating one would allow to approximate the other. This equation can also be used to show Scattershot BS is a special case of GBS, by directly writing the hafnians that appear in the description of GBS with the permanents that appear as the amplitudes of a SBS set-up [Krus19]. As noted in [Krus19], BS with Fock-state inputs but Gaussian detectors (e.g. homodyne detection) is also formally equivalent to a (time-reversed) version of a GBS set-up.

It is interesting to observe how different variations of BS lead to sampling from different probability distributions. In the original BS proposal, the probability amplitudes can be written as submatrices of the (general) unitary matrix describing the interferometer. GBS samples from hafnians of submatrices of a general symmetric matrices, which enables applications in graph theory, where the symmetric matrix is chosen as a graph's adjacency matrix. The more recent proposal of Bipartite GBS duplicates the size of the interferometer to be able to sample from permanents of submatrices of general complex matrices. This progressive extension of the set of matrices determining the experimental probability amplitudes opens ways to improve the computational complexity proofs, and at the same time may suggest different applications for these sampling machines.

Fock-state BS is hard to verify, by its very complexity. A physically motivated way to get some partial verification that a BS device is working as it should comes from the efficient classical simulability of Gaussian processes – one could simulate homodyne detection from Gaussian state inputs as a way to verify the device is working appropriately, prior to changing the set-up to instead include photodetection at the output. Despite not testing the photodetection part, this could increase our confidence that the linear-optical dynamics is working as expected.

3.3 Non-linear BS, Fock-state BS and computational expressivity

As we have already described, we can go beyond the linear-optics paradigm in photonic sampling machines, by studying the computational complexity of linear interferometers interleaved with non-linear photonic gates, as in the Non-linear BS proposal (NLBS) [Spag21]. With enough non-linear interactions, we know it is possible to perform general quantum computation – for example, by using the non-linear gates to grow cluster states, then applying adaptively chosen photonic circuits to drive the measurement-based computation. An interesting question to investigate is then how the computational expressiveness of the restricted photonic quantum computer increases as we expand its functionality using non-linear gates. Using linear optics and measurement-induced non-linearities it is possible to simulate simple non-linear gates, such as the non-linear phase shift gate. As discussed previously, we can then use the postselection overhead, as well as the classical computational cost of finding the optimal interferometer design, as ways to quantify the complexity of the non-linear gate implemented.

4 Concluding remarks

In this report we have reviewed several variations of photonic sampling machines. In common, they use non-interacting photons to generate distributions that are conjectured to be hard to sample from using classical computers.

Some variations, such as Scattershot Boson Sampling and Driven Boson Sampling, were proposed to make best use of probabilistic single-photons sources, for example those using the SPDC process. Gaussian Boson Sampling (GBS) and its variations result in a higher event rate, as all photons created by such non-linear processes can be input into the interferometer. Other variations propose photonic quantum interference in a bipartite interferometer, either to analyse the correlations that arise (Non-local Boson Sampling), or to enhance the distributions that can be sampled from (Bipartite Gaussian Boson Sampling). We have reviewed some of the known connections between the models, for example the way that BS using a SPDC source can be seen as a special case of GBS, Non-local Boson Sampling, and Bipartite GBS. We have also reviewed how some restrictions, such as having small interferometer depth, allow for more efficient classical simulation.

Other variations explore how the power of photonic quantum computation can be enhanced by a departure from the non-interacting photon picture, such as Non-linear BS. Interestingly, linear-optical simulation of these non-linear interferometers can be used to gain insights on the computational complexity of some non-linear interactions. As some kind of non-linearity is believed to be required for universal photonic quantum computation, this is an investigation relevant to the longer-term future of photonic quantum computation.

One common theme that we have reviewed is the search for a more expressive set of distributions to sample from. In going from BS to GBS, and then to Bipartite GBS, we went from sampling from permanents of submatrices of unitary, then symmetric, then arbitrary complex matrices. The study of the possible uses of such distributions, and how to achieve them experimentally, is one of the goals of project PHOQUSING.

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